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LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF A THIN, ELASTIC CIRCULAR PLATE WITH MIXED BOUNDARY CONDITIONS

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1. INTRODUCTION

The extremely important problem of vibration and buckling of a circular plate clamped on part of its boundary and simply supported on the remainder was approached by Bartlett four decades age [1]. In a very ingenious and rather pragmatic manner, he derived two variational principles for the lowest eigenvalue showing that the governing expressions can be determined by separation of variables. Upper and lower bounds for the fundamental frequency coefficient were shown to be close together. Later, Noble [2, 3] obtained an asymptotic expression which yields very good agreement with Bartlett's results.

The present research project arose from the need of knowledge of some higher natural frequencies. Furthermore, there is practically no information in the technical literature for the mixed boundary combination: clamped–free and simply supported–free. The lower natural frequency coefficients are determined in the present study for the three combinations of edge conditions (Figure 1): simply supported–clamped, simply supported–free, clamped–free. Accurate values of the eigenvalues are obtained using a well-known finite element code [4] and a dense net. The problem is of technological importance since it arises in the design of printed circuit boards, valve systems, transducers, etc.

2. FINITE ELEMENT RESULTS

The frequency coefficients have been determined using a net of 20232 elements; see Figure 2. The Poisson ratio was taken equal to 0·25 for the configuration shown in Figure 1(a) in order of verify the relative accuracy of the results obtained herein, as compared with the values presented in references [1, 2].

Very good agreement is observed when examining the results presented in Table 1 where additional frequency coefficients obtained by means of the finite element technique are shown. The first lower natural frequency coefficients are again determined for $\nu = 0{\cdot}30$ for the arrangement depicted in Figure 1(a) (see Table 2), while Tables 3 and 4 contain the natural frequency coefficients for the simply supported–free and the clamped–free arrangements respectively (see Figures 1(b) and (c)). The agreement with the exact eigenvalues is excellent when classical boundary arrangements are considered.

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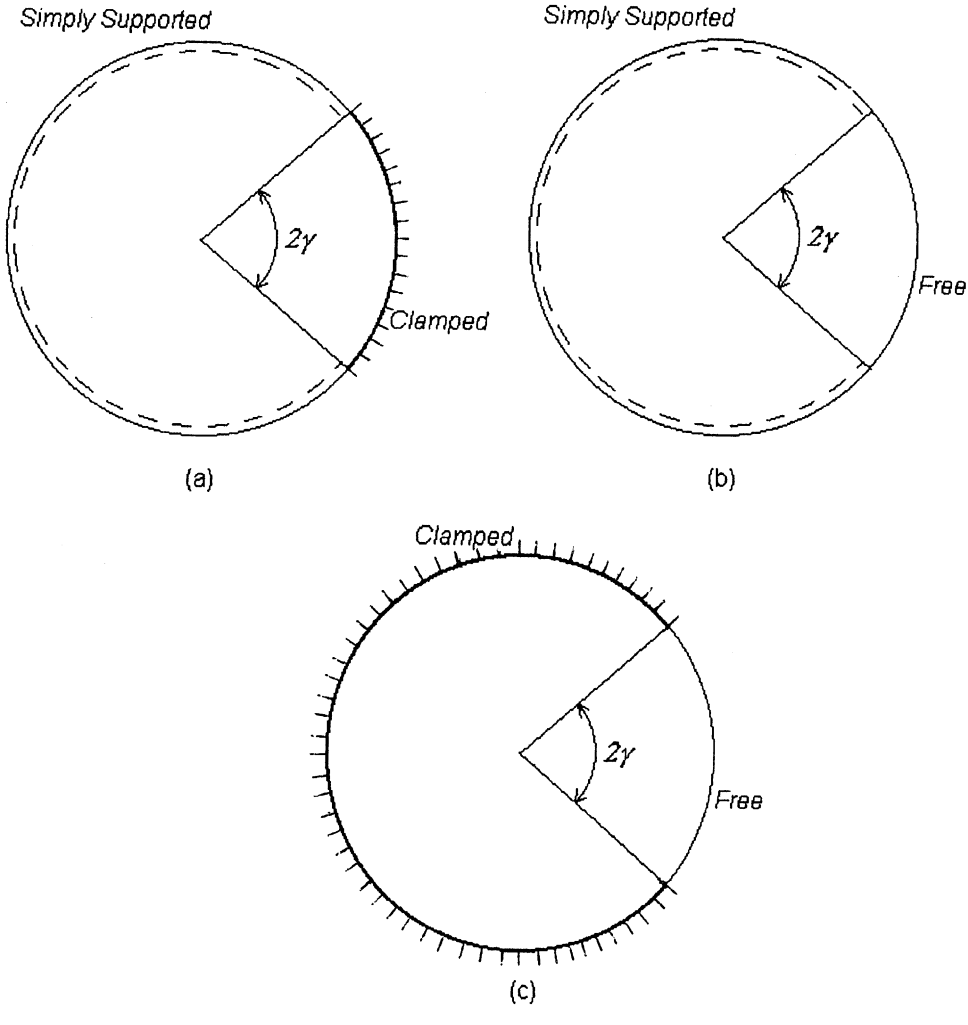

Figure 1. Vibrating structural element and different arrangements of boundary conditions: (a) simply supported–clamped, (b) simply supported–free and (c) clamped–free.

TABLE 1

Values of $\Omega_i = \omega_i a^2 \ (\rho h/D)^{1/2}$ *with* $\nu = 0{\cdot}25$ *in the case of a circular plate partially clamped* (2γ) *and partially simply supported* $(2\pi - 2\gamma)$

	γ								
	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$	π
Ω_1	4·868	5·859	6·351	6·878	7·511	8·272	9·118	9·880	10·216
Ω_2	13·842	14·070	14·748	15·886	17·485	17·590	18·276	19·980	21·262
Ω_3	13·842	15·939	16·904	17·404	17·550	19·493	20·833	21·232	21·262
Ω_4	25·557	26·331	27·128	28·004	30·217	30·690	31·704	32·930	34·878
Ω_5	25·564	27·054	28·122	29·829	30·277	31·603	32·873	34·697	34·882
Ω_6	29·669	31·585	33·051	33·771	34·504	36·621	38·134	38·902	39·775
Ω_7	39·907	41·338	42·109	43·223	45·490	46·536	47·552	49·009	51·037
Ω_8	39·907	41·523	42·965	44·554	45·705	46·895	48·639	50·485	51·037
Ω_9	48·432	48·653	49·989	52·206	54·153	54·664	56·264	58·202	60·837

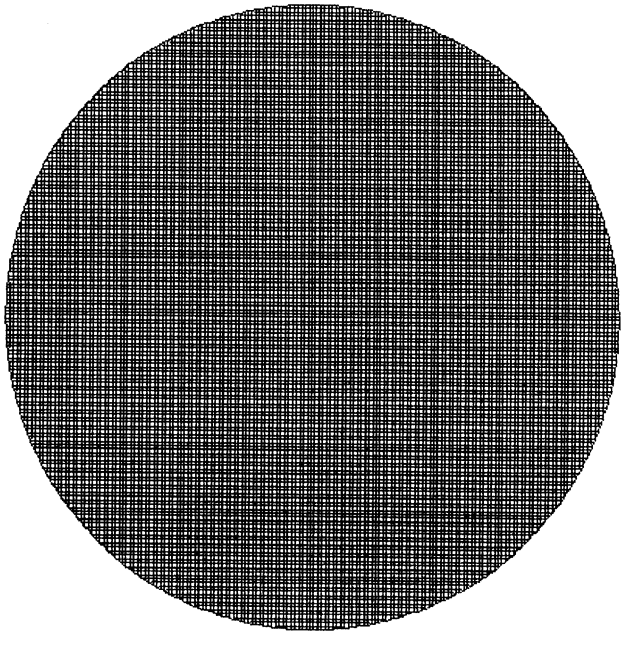

Figure 2. Finite element net used in the present investigation (20232 elements).

TABLE 2

Values of $\Omega_i = \omega_i a^2 \ (\rho h/D)^{1/2}$ *with* $\nu = 0{\cdot}3$ *in the case of a circular plate partially clamped* (2γ) *and partially simply supported* $(2\pi - 2\gamma)$

	γ								
	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$	π
Ω_1	4·943	5·915	6·401	6·922	7·546	8·296	9·130	9·882	10·216
Ω_2	13·905	14·130	14·802	15·931	17·514	17·618	18·300	19·988	21·262
Ω_3	13·905	15·982	16·939	17·434	17·582	19·508	20·836	21·232	21·262
Ω_4	25·616	26·384	27·179	28·048	30·247	30·716	31·726	32·943	34·878
Ω_5	25·623	27·106	28·165	29·860	30·303	31·625	32·886	34·698	34·882
Ω_6	29·727	31·628	33·085	33·803	34·534	36·636	38·140	38·906	39·775
Ω_7	39·965	41·387	42·155	43·264	45·519	46·559	47·570	49·020	51·037
Ω_8	39·965	41·572	43·007	44·587	45·731	46·916	48·653	50·487	51·037
Ω_9	48·488	48·708	50·036	52·242	54·186	54·691	56·282	58·213	60·837

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TABLE 3

Values of $\Omega_i = \omega_i a^2 (\rho h/D)^{1/2}$ *with* $\nu = 0{\cdot}3$ *in the case of a circular plate partially free* (2γ) *and partially simply supported* $(2\pi - 2\gamma)$

	γ								
	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$	π
Ω_1	4·943	4·924	4·592	3·047	1·782	1·032	0·584	0·261	0·000
Ω_2	13·905	13·574	8·675	6·455	6·309	4·195	2·834	1·890	0·000
Ω_3	13·905	13·903	13·672	10·672	6·555	6·221	5·458	4·190	0·000
Ω_4	25·616	23·782	16·649	15·715	13·948	9·733	8·149	7·869	5·358
Ω_5	25·623	25·588	22·512	16·456	15·488	13·908	10·525	7·978	5·358
Ω_6	29·727	28·811	27·318	23·379	17·325	17·064	15·873	12·294	9·002
Ω_7	39·965	34·491	29·966	27·996	24·004	18·983	17·903	16·561	12·437
Ω_8	39·965	39·973	31·688	28·530	27·672	22·482	18·669	17·668	12·437
Ω_9	48·488	43·207	41·304	33·322	30·836	30·039	23·661	20·654	20·470

TABLE 4

Values of $\Omega_i = \omega_i a^2 (\rho h/D)^{1/2}$ *with* $\nu = 0{\cdot}3$ *in the case of a circular plate partially free* (2γ) *and partially clamped* $(2\pi - 2\gamma)$

	γ								
	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	$5\pi/8$	$3\pi/4$	$7\pi/8$	π
Ω_1	10·216	9·871	8·008	4·225	2·455	1·600	1·143	0·855	0·000
Ω_2	21·262	19·682	10·978	9·775	7·196	4·573	3·091	2·092	0·000
Ω_3	21·262	21·230	20·584	12·455	9·320	8·443	6·422	4·784	0·000
Ω_4	34·878	29·720	21·198	20·573	14·865	10·400	9·354	8·249	5·358
Ω_5	34·883	34·687	26·356	21·169	19·850	15·354	11·090	8·946	5·358
Ω_6	39·775	37·306	35·047	25·659	20·939	20·783	16·804	12·723	9·002
Ω_7	51·037	41·219	36·036	34·272	25·411	21·021	20·026	17·256	12·437
Ω_8	51·037	50·387	39·050	34·974	32·777	23·586	20·853	20·084	12·437
Ω_9	60·837	52·295	49·899	39·345	35·201	32·665	24·007	20·740	20·470

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